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SIMULATION OF MOTION OF IONS IN THE CHANNEL OF A STATIONARY PLASMA ENGINE

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The results of numerical simulation of the process of motion of ions of a working mass in the accelerating channel of a stationary plasma engine SPD-70 on the basis of a direct numerical solution of the kinetic equation of rarefied plasma in an accelerating electrostatic field are presented.

Keywords: stationary plasma engine, kinetic equation of rarefied plasma, numerical simulation

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Introduction. Formulation of the research task

The modern trend of industrialization and commercial use of near-Earth space implies the launch and operation of space vehicles (SCs) of various purposes. Such vehicles can include a variety of spacecrafts, for example, Earth remote sensing satellites (RS), communication and navigation satellites in low (up to 500 km) orbits, navigational spacecraft in high circular orbits, space communication satellites in strongly elliptical orbits, geostationary satellites, etc.

The wide range of spacecraft and the variety of tasks that are solved with their use makes it necessary to create highly effective small-thrust propulsion systems designed to solve problems of attitude control and stabilization of spacecraft, maintaining orbital parameters, solving mission control tasks, i.e. transfer of spacecraft from a low Earth orbit to the target one, incl. geostationary.

A special place among low-thrust engines for spacecraft is occupied by electric rocket engines (ERE). In general, the fields of application of ERE are known: these are problems requiring the development of a significant total impulse, of the order of (~ 10^6 N•s). In this area, EREs are out of the competition, since electric rocket engines, having a specific impulse value of 1000 to 10000 s, are much more economical in terms of the consumption of the propellant than chemical fuel engines [1, 2]. Consequently, the mass of the charged electric rocket propulsion system (ERPS) is much less than that of systems based on liquid-fuel rocket engines (LREs), even taking into account a fairly large mass of both the engines themselves and the system for storing and supplying the propellant (SSSP) and the system conversion and control (SCC), which are part of the ERPS.

Among the various types of ERE, the most widely used are stationary plasma engines (SPE). The main idea used in SPE is the acceleration of ions in the accelerating channel by an electric field induced by a magnetic field in an extended plasma layer and implementation of focusing of accelerated ions by creating a focusing geometry of the electric field equipotentials using the appropriate geometry of the magnetic field lines and aligning the potential along magnetic lines of force due to relatively free movement of electrons along these lines [3].

The typical structure of the SPE shown in Fig. 1 [14] includes anode 1, cathode 2, dielectric (ceramic) discharge chamber 3, magnetic system with an external pole 4, magnetization coils 5, internal pole 6 and magnetic core 7. The output sections of the walls 8, 9 of the discharge chamber are usually located in the inter-polar gap.

An annular accelerating channel is located in the discharge chamber, in the depth of which there is an annular anode serving for feeding and uniform distribution of the flow of the propellant gas (for example, xenon) in the accelerating channel. The cathode is located outside the accelerating channel and is a gas-discharge source of electrons based on a hollow cathode.

In the accelerating channel, a magnetic system is used to create a predominantly radial magnetic field as well as a predominantly longitudinal electric field by applying a constant voltage between the anode and the cathode. In an electric discharge in crossed electric and magnetic fields, which is maintained between the anode and the cathode in the gas flowing in the accelerating channel, it is possible to efficiently ionize the atoms of the propellant entering the accelerating channel through the anode-gas distributor, and also to accelerate the formed ions by the electric field, created in the area of the accelerating channel.

Atoms of the propellant are ionized by electrons entering the discharge from the cathode and those newly formed in the discharge. The accelerated flow of ions comes out from the accelerating channel and is neutralized by a part of the electrons entering it from the cathode. In the process of ion acceleration, a reactive thrust is created, which is used for controlling the spacecraft.

Design, development and optimization of electric rocket engines involve a large amount of experimental research, including using vacuum equipment, which significantly increases the time spent on the development of ERE. These problems can be partially solved by applying mathematical modeling of the physical processes taking place in the components and subsystems of the ERE. For stationary plasma engines, this is primarily the processes of ionization of the atoms of the propellant in the accelerating channel, the steady flow of ions in the channel, the processes of their acceleration and focusing, and also the processes of neutralization of the ion beam at the output of the SPE.

At present, many studies have been devoted to the modeling of processes in the accelerator channel of SPEs (see, for example, [4-11]). Some researchers use algebraic relations in the construction of models of processes in the SPE [7, 8, 10], others, considering kinetic equations, use various versions of the particle method for their solution [4, 5, 6, 9, 11].

The purpose of this study is development and verification of the computational model of the steady-state ion flow in the accelerating channel of the SPE on the basis of a direct numerical solution of the kinetic equation of rarefied plasma.

The Computational Model of Ion Flow in the SPE Channel

As the basis for constructing the ion motion model in the SPE channel, the kinetic equation of ion motion in an electrostatic field was used [12]:

$$\begin{gathered} f_t + \mathbf{c}\nabla_{\mathbf{r}} f + \mathbf{F}_a \nabla_{\mathbf{c}} f = I_{coll}, \\ \mathbf{F}_a = \frac{e}{m}\mathbf{E} + e\mathbf{c} \times \mathbf{B} + \mathbf{F}_{ab} \end{gathered} \tag{1}$$

Here $f = f(t, \mathbf{r}, \mathbf{c})$ is the density of the ion distribution function over the space and velocities, $\mathbf{r}, \mathbf{c}$ are respectively the radius vector and the velocity of the particle, $\nabla_{\mathbf{r}}, \nabla_{\mathbf{c}}$ are the gradient operators in the corresponding subspaces of the phase space, I_{coll} is the term taking into account the collisions of ions with each other and with other particles (electrons and neutral atoms of the propellant), $\mathbf{F}_a$ is the resultant of external forces acting on ions, $\mathbf{E}, \mathbf{B}$ are the strengths of the electrostatic and magnetic external fields, e, m are the charge and mass of the ion, and $\mathbf{F}_{ab}$ are the long-range force from other charged particles in the flow.

Equation (1) is a nonlinear integro-differential equation with respect to an unknown function of seven variables. Its solution is a very difficult task, so the following simplifying assumptions were made:

- the ion flow in the accelerating channel was assumed to be stationary
$$f_t = 0$$
- the influence of the magnetic field and the interaction with other particles on the motion of the ions was assumed to be small in comparison with the influence of the electrostatic field
$$I_{coll} = 0, \quad \mathbf{B} = 0, \quad \mathbf{F}_a = 0$$
- the electrostatic field changes only along the axis of the accelerating channel of the SPE
$$\mathbf{E} = \mathbf{i}E.$$

The first of the assumptions is stipulated by the goal posed in this study, the second one is that the mean free path of ions in the SPE channel is comparable to the size of the channel itself (the Knudsen number $Kn = \frac{l}{L} \sim 1$), the third follows from the design of the SPE. Taking these assumptions into account, equation (1) takes the following form:

$$\mathbf{c}\nabla_{\mathbf{r}} f + \frac{e}{m} E f_c = 0$$

Finally, after averaging the last expression along the cross section of the accelerating channel of the SPE, the following equations were obtained describing the motion of the ions of the propellant in the accelerating channel of the SPE in the steady-state mode:

$$c f_x + \frac{e}{m} E f_c = 0 \tag{2}$$

The obtained equation is much simpler than the original equation (1): equation (2) is a linear hyperbolic equation in partial derivatives depending on a function of two variables $f = f(x, c)$ (here x, c are the coordinate relative to the axis of the accelerating channel and the projection of the particle velocity on this axis, respectively).

The solution of equation (2) is to be sought in the following calculation area:

$$S = \left(x \in [0, l] \cup c > 0\right) \tag{3}$$

Here l is the length of the accelerating channel. To complete the formulation of the problem, we should specify the boundary conditions. Since the coefficients of equation (2) are positive values, it is sufficient to give the values of the density distribution function of the ions on the ray $\left(x=0\cup c>0\right)$. and segment $\left(x\in\left[0,l\right]\cup c=0\right)$.

On the ray $\left(x=0\cup c>0\right)$, the following Maxwell distribution was specified

$$\left(x,c\right)\in\left(x=0\cup c>0\right)\quad\Rightarrow\quad f\left(c,x=0\right)=f_m=\frac{\rho_{in}}{\sqrt{2\pi RT_{in}}}e^{-\frac{(c-V_{in})^2}{2RT_{in}}}. \qquad (4)$$

Here $\rho_{in}, V_{in}, T_{in}$ are, respectively, the macroscopic density, the velocity and temperature of the propellant at the entrance to the accelerating channel, is the gas constant of the propellant.

On the segment $\left(x\in\left[0,l\right]\cup c=0\right)$, the zero value of the density of the distribution function was set:

$$\left(x,c\right)\in\left(x\in\left[0,l\right]\cup c=0\right)\quad\Rightarrow\quad f\left(c=0,x\right)=0. \qquad (5)$$

To build a computational model, the estimated area (3) was discretized by constructing a uniform grid

$$W_{N,M}=\left\{\left(c_n,x_i\right)\middle|n=0,1,\ldots,N;i=0,1,\ldots,M\right\}$$

Here $c_n=nh_c, x_i=ih_x$ are the values of the independent variables in the grid nodes, h_c, h_x are the steps of the discretization of the respective coordinates. For the numerical solution of equation (2) with closing statements (3, 4) on the grid $W_{N,M}$, the marching method with velocity c as the march coordinate. Equation (2) was solved by means of a difference scheme of the third order of accuracy with respect to both independent coordinates obtained by the method of joint approximation [13]:

$$\frac{1-Cu}{\left(2+Cu\right)}f_{i-1}^{n+1}+f_i^{n+1}=\frac{1+Cu}{2}f_{i-1}^{n}+\left(1-Cu\right)f_i^{n}+\frac{Cu\left(1-Cu\right)}{2\left(2+Cu\right)}f_{i+1}^{n} \qquad (6)$$

Here $f_i^n=f(c_n,x_i)$ is the value of the desired function at the nodes of the calculated grid, and $Cu=\frac{h_c}{h_x}\frac{e}{c_n m}E$ is the Courant number of the problem (5). We note that the difference scheme (6) is stable when $Cu<1$

Results of Calculation of Ion Flow Parameters in the Accelerating Channel of SPD-70

With the use of the computational model presented in the previous section of this work, the modeling of the flow of ions of a working medium in the SPD-70 thruster was carried out. The main geometric parameters of the computational domain are shown in Fig. 2.

As parameters of the propellant, those of xenon were considered, the mass flow was assumed to be 2.8 mg/s, the potential difference in the accelerating channel was 100V. The gas parameters at the entrance to the accelerating channel were taken from the solution of the problem of gas flowing into vacuum:

$$\rho_{in} = 3.68\ \frac{мкг}{м^3}, \quad V_{in} = 310\frac{м}{с}, \quad T_{in} = 110K.$$

The solution of problem (2, 4, 5) was carried out in dimensionless variables. As the length scale, the length of the accelerating channel was chosen, the time scale was assumed equal to 0.01 ms. With this choice of scales, when numerical simulation is performed, the effect of rounding errors on the results obtained is excluded. In the computation grid, 100 nodes were selected for the spatial variable. The relationship of the parameters of the problem (2, 4, 5) made it possible to choose the discretization step along the march coordinate equal to the space step, since the Courant number in this case will be much less than unity and will decrease when moving along the march coordinate. The calculation was carried out until the value of the density of the distribution function became zero along the entire length of the acceleration channel. Note that this problem with such a formulation was solved in several seconds by a personal computer of a moderate computing power.

Figure 3 shows the density distribution function of xenon ions in the accelerating channel of the ESE SPD-70, obtained in calculations using the computational model described above. As can be seen from the plot, the form of the distribution function corresponds to the existing ideas about the nature of the processes in the accelerating channel of the SPE: the center of the distribution shifts towards higher velocities (this indicates an increase in the macroscopic velocity of the ion flow), and the variance decreases (indicating a decrease in the thermodynamic temperature).

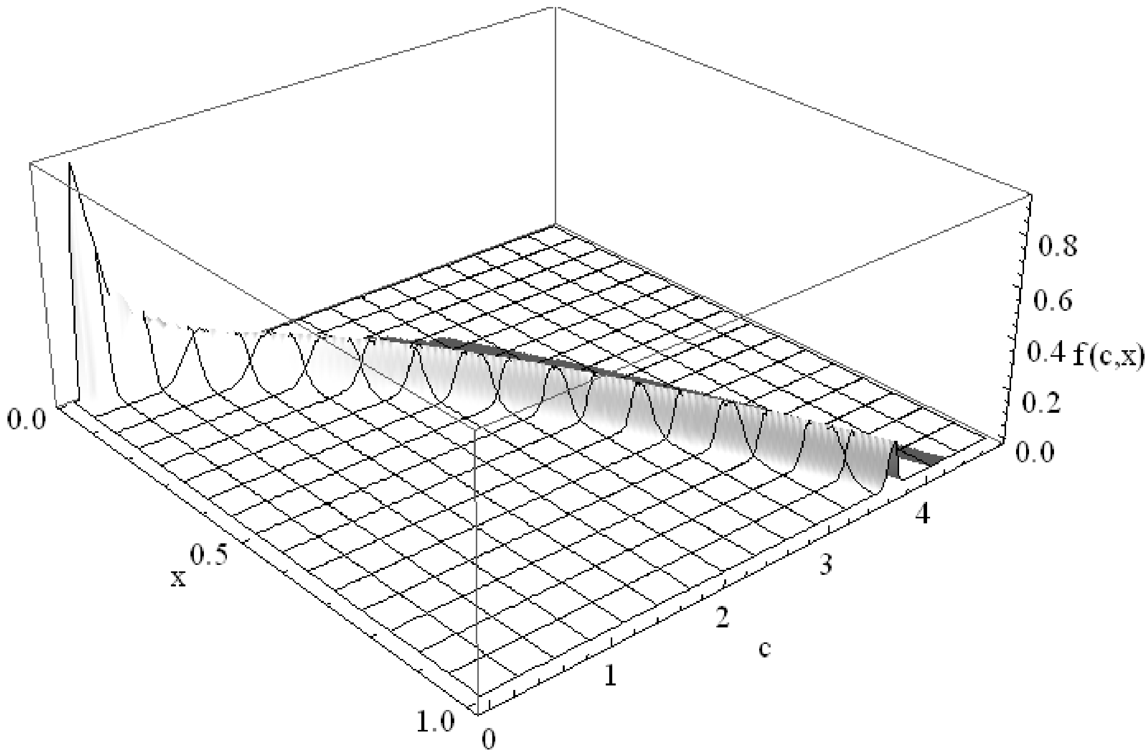

Fig. 3. The graph of the density distribution function of xenon ions in the accelerating channel

For carrying out the quantitative analysis of the obtained results, the first moments of the velocity distribution function corresponding to the macroscopic ion flux parameter were taken:

$$\rho(x) = M_c^1(f(c,x)) = \int_{-\infty}^{\infty} f(c,x)dc$$

$$\rho u(x) = M_c^2(f(c,x)) = \int_{-\infty}^{\infty} f(c,x)cdc$$

Here ρ is the macroscopic density of the ion flux, ρu is the kinetic momentum.

Basing on the known first moments, the main flow parameters in the accelerating channel and the thrust of the ERE SPD-70 were determined from the dependencies given below:

$$\dot{m}(x) = \rho u(x)F, \quad V(x) = \frac{\rho u(x)}{\rho(x)}, \quad P = \dot{m}(x)V(x)\big|_{x=l}.$$

Here $\dot{m}(x)$ is the mass flow rate of the ions of the propellant, $V(x)$ is the macroscopic velocity of the ions, P is the thrust of the ERE. In Fig. 4 for the sake of illustration, the graph of the change in the ion flow velocity along the accelerating channel is given. As can be seen, in accordance with the ideas about the processes in the accelerator channel of the SPE, the ion velocity increases. The deviation of the mass flow from the nominal value of 2.8 mg/s along the accelerating channel did not exceed 3%, which indicates the acceptable quality of the results obtained by the proposed computational model. Finally, the numerical values of the parameters of the ERE SPD-70, obtained from the calculation results (the ion velocity at the exit from the accelerating channel $V(x=l) = 15.05 \,{}^{км}\!/_{c}$ and the thrust of the ERE $P = 41.67 мН$) are in good agreement with the experimental data ($V = 14.7 \,{}^{км}\!/_{c}, P = 40 мН$, according to the results given in [14]). The overestimated values of the calculated parameters are due to the fact that the process of ionization of the atoms of the propellant was not taken into account in the model.

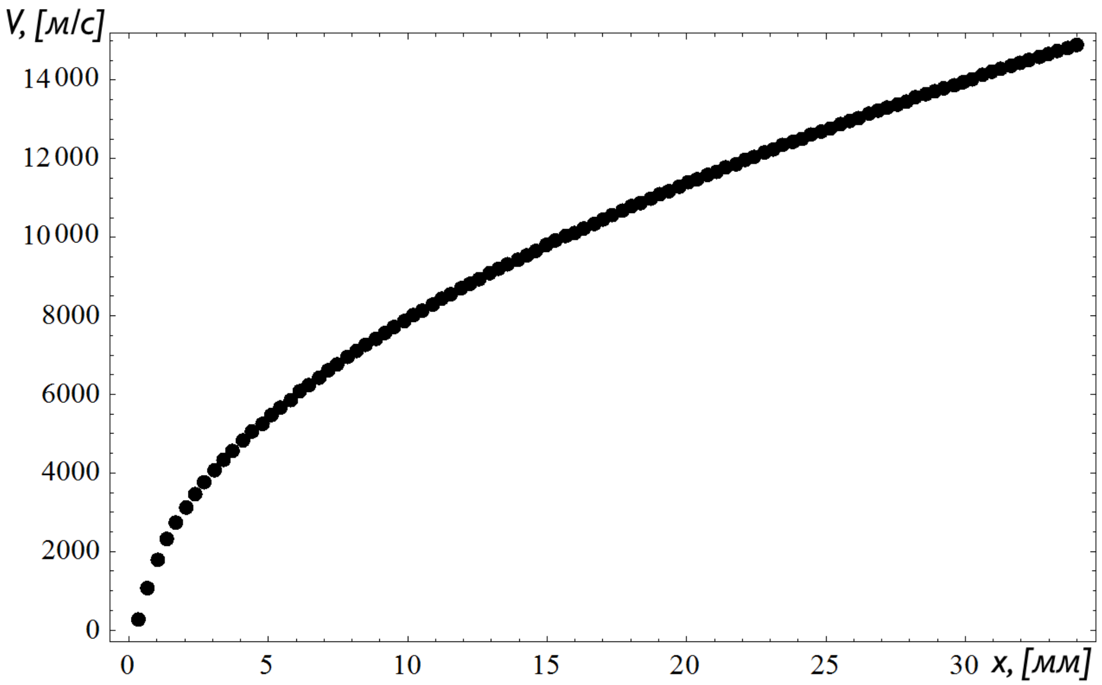

Fig. 4. Change of the velocity of the ion flow along the accelerating channel.

Conclusions and Direction of Further Research

As a summary, the following conclusions can be drawn:

1) In this paper we propose a new computational model of ion motion in the accelerating channel of the stationary plasma thruster, based on the direct solution of the kinetic equation for the motion of charged particles in the electrostatic field for large values of the Knudsen number.

2) Calculations of the movement of ions of the propellant in the accelerating channel of the ERE SPD-70, which required an insignificant time of computer processing.

3) The results of the calculations are in good agreement with modern ideas about the processes in the SAP and the available experimental data.

All of the above allows us to recommend the proposed approach for investigating processes in a rarefied plasma and in the designing of an ERE.

In the future, it is proposed to develop the described approach in the following directions:

1) To take into account the influence of the magnetic field on the motion of ions and the interaction of particles.

2) To construct a computational model of the motion of electrons in the accelerating channel of the SPE.

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