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ALGORITHMS OF AUTOMATIC MONITORING OF PARAMETERS OF ELECTRIC ROCKET PROPULSION SYSTEMS

Algorithms of automatic monitoring of the parameters electric propulsion systems are considered. The algorithms for detecting jump-like changes in informative signs for different volumes of apriority information and changes of the information signs in a form of the linear trends are presented.

Keywords: electric propulsion system, measurements of the parameters, automatic monitoring, algorithms of monitoring.

It is known that the parameters and characteristics of electric rocket propulsion systems (ERPS) vary with time [1, 3]. Hence follows the problem of automatic monitoring of parameters of subsystems of ERPS in the course of their functioning. For automatic monitoring of parameters of subsystems of ERPS, measurements of informative signs/indicators that characterize the functioning and operability of the system are used.

The model of such measurements can be a sequence of random variables - a one-dimensional random signal X(1), X(2), X(3), ..., X(k) or a sequence of random vectors [X(1)], [X(2)], ..., [X(k)], a multidimensional discrete random signal, where $[X(k)]=[X1(k), X2(k), ..., Xm(k)]^T$; where T is the sign of transposition. Properties of discrete signals and their use for solving forecasting and control tasks are discussed in [2]. It is obvious that the measured signals contain information on the state of the subsystems of the ERPS, and can be used to solve the monitoring tasks.

1. The problem of detecting the jumps of informative signs of subsystems of the ERPS under normal measurements in the conditions of uncertainty of the initial data.

Suppose that for some informative indicators characterizing the state of the propulsion system, only the mathematical expectation a1 and variance σ^2 are known. We can write the decision rule as:

$$|Z(k)| \geq Z_0,$$

where Z(k) is calculated by the formula

$$Z(k)=\frac{1}{n}\sum_{i=1}^{n}(X(k-i)-a_1).$$

It is obvious that the mathematical expectation M[Z(k)/0] in the interval 1≤k<k1 is zero, and the variance $D[Z(k)]=(1/n)\sigma^2$. We can determine the probability of a false alarm

$$P_{\square}=1-\int_{-Z_0}^{Z_0}W(Z/0)dZ=2\left(1-\varPhi\left(\frac{Z_0}{\sqrt{D}}\right)\right). \qquad (1)$$

From (6.9) follows the expression for calculating the threshold

$$Z_0=\frac{\sigma}{\sqrt{n}}\varPhi^{(-1)}\left(1-\frac{P_{\square}}{2}\right). \qquad (2)$$

On the section k≥(k1+n-1), the mathematical expectation Z(k) is equal to

$$M[Z\ (k)]=a_2-a_1=\Delta a.$$

Therefore, to calculate the probability of making a decision about the existence of a jump P_{pc}, we will have the formula

$$P_{pc} = 1 - \int_{-Z_0}^{Z_0} W(Z)dZ = 1 - \Phi\left[q\sqrt{n} + \Phi^{(-1)}\left(1 - \frac{P_{\square}}{2}\right)\right] + \\ + \Phi\left[q\sqrt{n} - \Phi^{(-1)}\left(1 - \frac{P_{\square}}{2}\right)\right], \tag{3}$$

where q=Δa/σ.

If the variance D[Z(k)] of the informative indicator is unknown, then it can be replaced by an estimate that can be obtained in the form

$$D^*[Z(k)] = \frac{1}{n^2}\sum_{i=1}^{n}\left(X(k-i) - a_1\right)^2 . \tag{4}$$

The threshold can be represented as

$$Z_0^*(k) = \sqrt{D^*(k)}\ \Phi^{(-1)}\left(1 - \frac{m_{\square}}{2}\right) \tag{5}$$

Fig. 1 shows the results of the computational experiment: initial signals X(k), Z(k), the threshold $Z_0(k)$ and the probabilities of the decision-making $P_{pc}(k)$ for the two cases: DZ(k)=σ^2/n and DZ(k)=D*(k). As can be seen from the presented graphs, the lack of information on dispersion has little effect on the detection of a jump.

If the parameters of the informative signs a_1 and a_2 are unknown, the measurements of X(k) are equal to X(k)=a_1+ΔX(k), k<k_1, X(k)=a_2+ΔX(k), k≥k_1; The moment k 1 and the direction of the jump (up or down) are unknown, then the above-considered measurement processing algorithms cannot be used. Therefore, we divide the window in half and select two samples:

X(k-n/2), X(k-n/2 +1), ... , X(k-1);
X(k), X(k+1), ... , X(k+n/2 - 1).

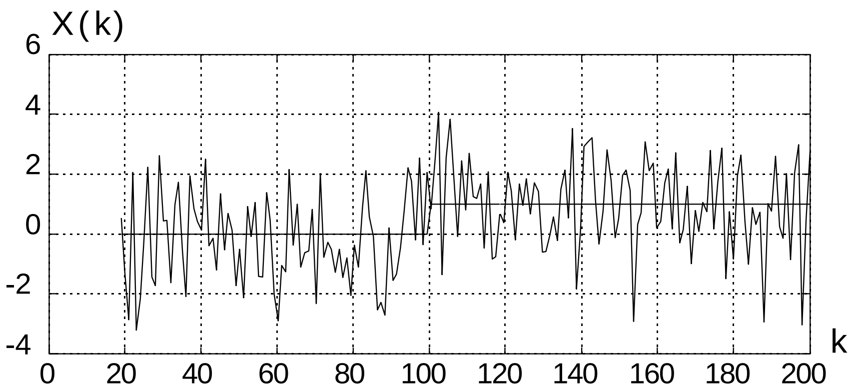
We determine the mean values for each sample of measurements

$$y_1(k) = \frac{2}{n}\sum_{i=1}^{n/2} X(k-i),\ \ y_1(k) = \frac{2}{n}\sum_{i=1}^{n/2} X(k+i-1).$$

The difference between the obtained mean values

$$Z(k)=y_2(k) - y_1(k)$$

can serve as an indicator of the homogeneity of the controlled sample X (k). In the absence of a jump, the mathematical expectation Z(k) is zero, and the variance is (4/n)σ^2.



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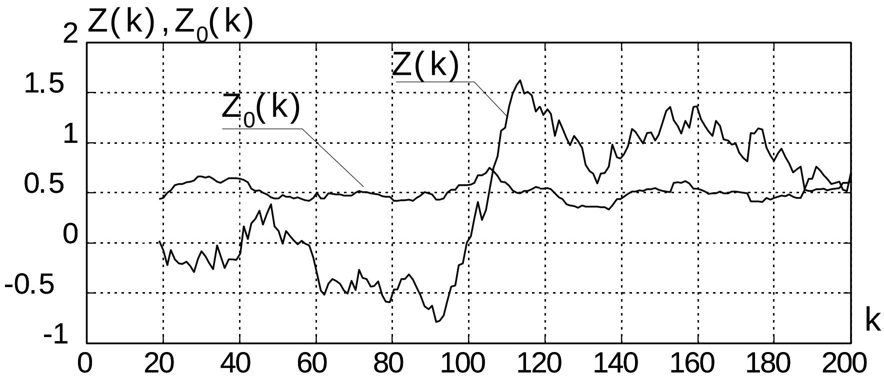

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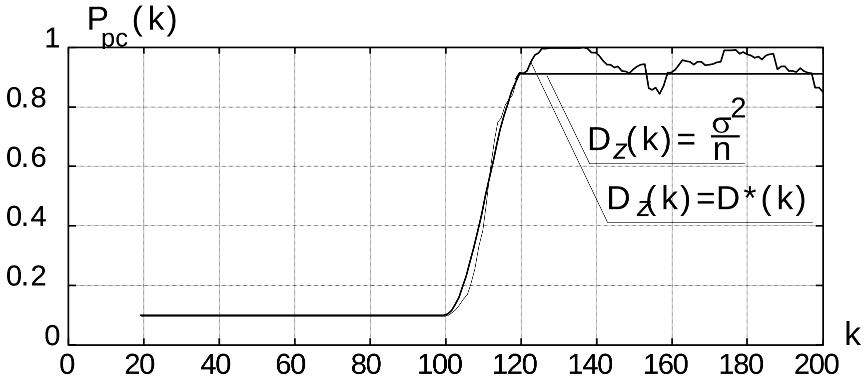

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Fig. 1. Outcomes of modeling the process of detection of an abrupt change of the signal (a_1 и σ^2 are unknown)

If we use the decision rule in the form $|Z(k)| \geq Z_0$, then

$$m_{\square} = 2\left[1 - \Phi^{(-1)}\left(\frac{Z_0\sqrt{n}}{2\sigma}\right)\right].$$

From this expression we define the threshold value

$$Z_0 = \frac{2}{\sqrt{n}}\sigma\Phi^{(-1)}\left(1 - \frac{m_{\square}}{2}\right).$$

The probability of making a decision about the presence of a jump in Ppc depends on the expectation Z(k), the value of which changes in the transition zone (k1 - n/2) ≤ k ≤ (k1+n/2 - 1) according to the law of an isosceles triangle

$$M_2(k) = \begin{cases} \Delta a + \frac{2\Delta a}{n}(k - k_1), & (k_1 - \frac{n}{2}) \leq k \leq k_1, \\ \Delta a - \frac{2\Delta a}{n}(k - k_1), & k_1 \leq k \leq (k_1 + \frac{n}{2}). \end{cases} \tag{6}$$

Consequently, in this zone

$$P_{pc}(k) = 1 - \Phi\left[\frac{M_2(k)\sqrt{n}}{2} + \Phi^{(-1)}\left(1 - \frac{m_{\square}}{2}\right)\right] +$$

$$+ \Phi\left[\frac{M_2(k)\sqrt{n}}{2} - \Phi^{(-1)}\left(1 - \frac{m_{\square}}{2}\right)\right].$$

At the moment k = k 1, the mathematical expectation $M_2(k)$ is maximal and equal to Δa. At the same time, the maximum value and probability Ppc (k)

$$P_{pc}(k)=1-\Phi\left[\frac{q\sqrt{n}}{2}+\Phi^{(-1)}\left(1-\frac{m_{\square}}{2}\right)\right]+ \\ +\Phi\left[\frac{q\sqrt{n}}{2}-\Phi^{(-1)}\left(1-\frac{m_{\square}}{2}\right)\right]. \quad (7)$$

If the variance of the measurement noise is unknown, then it can be replaced by an estimate:

$$D_Z^{*}(k)=\frac{4}{n^2}\left[\sum_{i=1}^{n/2}\left((X(k-i)-y_1(k))^2+(X(k+i-1)-y_2(k))^2\right)\right]. \quad (8)$$

For the threshold of comparison, we get

$$Z_0^{*}(k)=\sqrt{D_Z^{*}(k)}\Phi^{(-1)}\left(1-\frac{m_{\square}}{2}\right). \quad (9)$$

Fig. 2 shows the results of a numerical experiment on the detection of a jump with respect to the difference of means, if a_1, a_2 and σ^2 are unknown.

1. **The problem of detecting linear and quadratic trends in monitored parameters of electric rocket propulsion systems.**

Along with jump-like changes in the parameters of monitored processes in electric rocket propulsion systems, their slow changes are possible [6 - 8]. As a model of such processes can serve a linear trend, which can be represented in the form

$$U(k)=a0+a_1k. \quad (10)$$

Slow changes in monitored parameters can develop into rapid ones. In this case, they can be described by a quadratic equation of the form

$$U(k)=a_0+a_1k+a_2k^2. \quad (11)$$

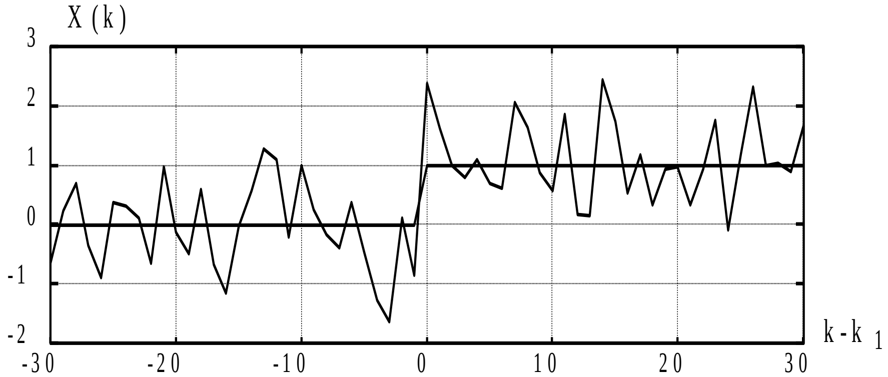
The processing of measurements of such monitored processes distorted by noise interference consists of detection of the beginning of a linear or quadratic trend and evaluation of its parameters. The model of the meter in this case is written in the form

$$X(k)=U(k)+\Delta X(k).$$

We assume that $\Delta X(k)$ is a sequence of normal random variables with zero mathematical expectation. In papers [4,5], various criteria for detecting trends are considered. It is assumed that the best of these is a criterion based on the rank correlation Ri between the order of random variables in time and their order in amplitude.

Consider a sample of random variables within the movable window

$$X(k-n+1),\ X(k-n+2),\ \ldots,\ X(k-j),\ \ldots,\ X(k-1),\ X(k),\ (j=1,2,\ldots,n).$$



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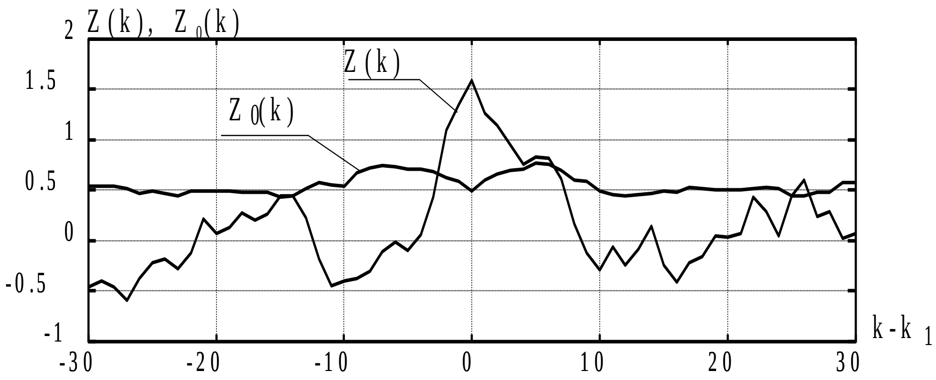

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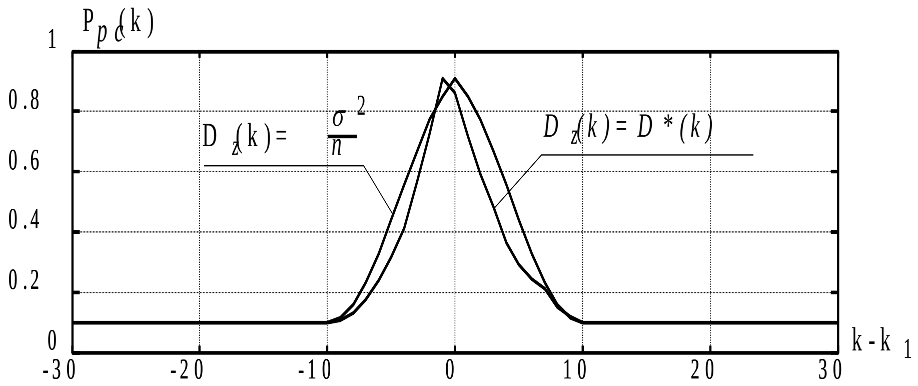

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Fig. 2. The results of modeling the process of detection of an abrupt change in the signal (a_1, a_2, σ^2 are unknown)

Let us introduce this function

$$\mathrm{H}_{ij}(k)=\begin{cases}1, & X(k-i+1)\geq X(k-j+1), i<j,\\ 0, & X(k-i+1)<X(k-j+1), i<j.\end{cases}$$

And calculate the number of pairs for which $X(k-i+1)\geq X(k-j+1), i<j$,

$$Q(k)=\sum_{i<j}^{n}\mathrm{H}_{ij}(k). \tag{12}$$

The last expression can be written in the form

$$Q(k)=\frac{1}{2}\sum_{i<j}^{n}\left[1+\operatorname{sign}(X(k-i+1)-X(k-j+1)\right]. \tag{13}$$

If the difference X(k-i+1)-X(k-j+1)=Z_{ij} is a random variable with zero expectation and a symmetric distribution law, then the mathematical expectation of Q(k) is

$$M\left[Q(k)\right]=\frac{n(n-1)}{4},$$

since the expectation is M[sign(X(k-i+1)-X(k-j+1))]=0.

As shown in [5], the variance of Q (k) depends only on the size of the window

$$D\left[Q(k)\right]=\frac{n(n-1)(2n+5)}{72}.$$

The ratio Q (k)/M[Q(k)] characterizes the degree of connection between the random variables X(k-i+1) and X(k-j+1). The coefficient of rank correlation is written in the form

$$r(k) = 1 - \frac{4Q(k)}{n(n-1)}. \qquad (14)$$

If there is no trend and X(k) are independent random variables with a symmetric distribution, then the probability density W(r) is approximated by a normal law with zero expectation and variance

$$D(r) = \frac{2(2n+5)}{9n(n-1)}.$$

If there is a trend within the window X(k)=a0 +a_1k +ΔX(k), the expectation $M[H_{ij}(k)]$ is equal to the probability that $Z_{ij}=(X_i - X_j) \ge 0$.

If X (k) are normal random variables, then Z_{ij} has a normal distribution with the expectation $M[Z_{ij}]=a_1(j-i)$ and variance $D[Z_{ij}]=2\sigma_x^2$

Consequently,

$$M\left[H_{ij}(k)\right] = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{2\sigma_x^2}} \int_0^{\infty} e^{-\frac{(Z-a_1(j-i))^2}{4\sigma_x^2}} dZ = \Phi\left[\frac{a_1(i-j)}{\sqrt{2}\sigma_x}\right],$$

$$M\left[Q(k)\right] = \sum_{i<j}^{n} M\left[H_{ij}(k)\right].$$

As a result, we obtain

$$M\left[Q(k)\right] = \sum_{i<j}^{n}\left[1 - \Phi\left(\frac{a_1(j-i)}{\sqrt{2}\sigma_x}\right)\right].$$

This expression can be converted to the form

$$M\left[Q(k)\right] = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left[1 - \Phi\left(\frac{a_1(j-i)}{\sqrt{2}\sigma_x}\right)\right] =$$

$$= \sum_{i=1}^{n-1}\sum_{j=1}^{n-i}\left[1 - \Phi\left(\frac{a_1 j}{\sqrt{2}\sigma_x}\right)\right] =$$

$$= \frac{n(n-1)}{2} - \sum_{i=1}^{n-1} i \cdot \Phi\left(\frac{a_1(n-i)}{\sqrt{2}\sigma_x}\right).$$

Thus, in the presence of a linear trend, the rank correlation coefficient is a normal random variable with mathematical expectation

$$a_r = M\left[r_k\right] = \frac{4}{n(n-1)}\sum_{i=1}^{n-1} i \cdot \Phi\left(\frac{q}{\sqrt{2}}\left(1 - \frac{i}{n}\right)\right) - 1, \qquad (15)$$

where q = na_1/σ_x is the signal-to-noise ratio (na_1 is the signal change due to the trend within the movable window).

If decisions are made with the Neumann-Pearson criterion

$$|r(k)| \ge r0,$$

then the threshold value is determined from the set value of the probability of false alarm

$$r_0 = \sqrt{D_r} \cdot \Phi^{(-1)}\left(1 - \frac{p_{\cdot}}{2}\right) = \sqrt{\frac{2(2n+5)}{9n(n-1)}} \cdot \Phi^{(-1)}\left(1 - \frac{p_{\cdot}}{2}\right). \qquad (16)$$

The probability of detecting a trend can be estimated with the formula

$$P_{\cdot} = 1 - \Phi\left(\frac{a_r + r_0}{\sqrt{D_r}}\right) + \Phi\left(\frac{a_r - r_0}{\sqrt{D_r}}\right). \qquad (17)$$

For $q \to \infty$, $a_r \to 1$, we can estimate the limiting possibilities of detecting trends

$$P_{p} = 1 - \Phi\left(\frac{3\sqrt{n(n-1)}}{\sqrt{4n+10}} + \Phi^{(-1)}\left(1-\frac{P}{2}\right)\right) +$$
$$+ \Phi\left(\frac{3\sqrt{n(n-1)}}{\sqrt{4n+10}} - \Phi^{(-1)}\left(1-\frac{P}{2}\right)\right).$$

Fig. 3 shows the limiting detection characteristics, from which it follows that for a probability of false alarm $P_{lm} > 0.1$, the size of the movable window should be chosen at least 10 ($n \geq 10$), and for a false alarm probability $P_{lm} > 0.001$, not less than 20 ($n \geq 20$). Dependence of the probability of detecting trends on the signal-to-noise ratio is shown in Fig. 4, which implies that for a reliable detection the following condition must be satisfied: q > (3..5).

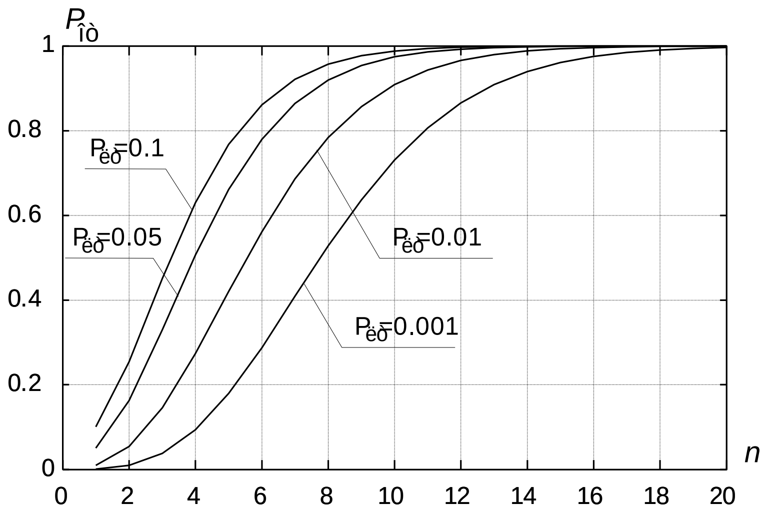

Fig. 3. Limiting characteristics of detection of trends in measured signals of electric rocket propulsion systems.

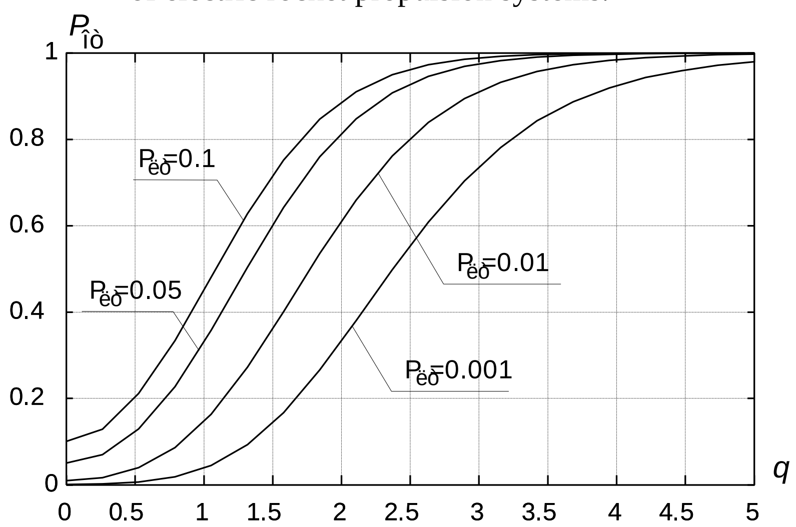

Fig. 4. Characteristics of detection of trends (n = 20).

Conclusions: The algorithms for automatic monitoring of variables characterizing the current state of electric rocket propulsion systems were considered. Algorithms are proposed that ensure the detection of an abrupt change in the informative indicators of subsystems of the ERPS, as well as changes in informative indicators in the form of trends. The proposed algorithms for automatic monitoring can be used in the development of various types of electric rocket propulsion systems.

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