A MATHEMATICAL MODEL OF HALL-EFFECT THRUSTER

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1. Analysis of experimental data the model is based on

In the course of experimental research of characteristics and stationary plasma thruster (SPT) parameters, one of the goals was to find optimal modes of its operation that would be characterized by the minimal average value of the discharge current and the minimal amplitude of oscillations at a given mass flow of the propellant [1, 2]. This ensures minimal energy expense on creation of the thrust and reduces the requirement to the power of the discharge power supply unit. Preliminary experiments showed that the determining factors in achieving the optimal operational modes are the choice of the current in the electromagnet, as well as the choice of structure and optimal parameters of the output filter of the discharge power supply unit. The results of the experimental research of an SPT100 thruster, corresponding to the optimal structure and parameters of the output filter and put into the foundation of the synthesis of a mathematical model of the SPT, are shown in Fig. 1 [2]. This figure shows dependences of the amplitude of the discharge current oscillations I_p and the average value of the discharge current $\bar{I}_d$ on the value of the current I_m in the electromagnet at different values of V_p. The dependence is extremal in its nature, which is preserved over different operational modes of the thruster and the change of the structure and parameters of the output filter of the power supply source.

Along with the determination of the static parameters of the SPT100 thruster, its dynamic characteristics were determined, which were obtained in the form of spectra of oscillations of the discharge current at different values of the current in the electromagnet [3]. The obtained spectra characterize the alternating component of the discharge current $\tilde{I}_d$. The frequencies of the registered oscillations are in the range from 1.0 to 400 kHz. The main power of oscillations lies in the range of up to 120 kHz, where the main peaks of the discharge current oscillation spectrum are located. With the decrease of the current in the electromagnet from 4.5 A to 2.6 A, the amplitude of the peaks sharply decreases (by 20 dB).

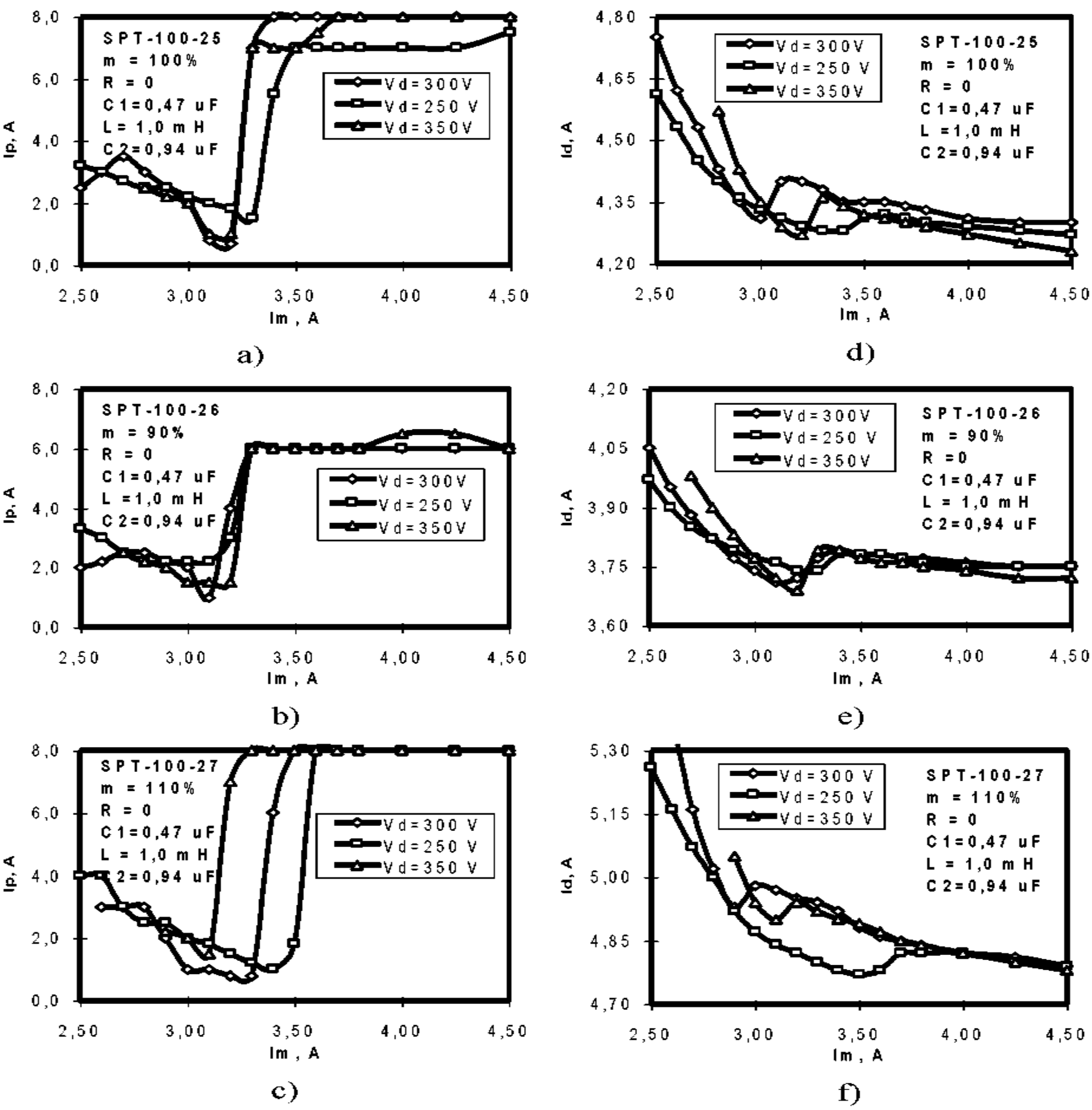

Fig. 1. Dependence of the amplitude of oscillations I_p and the average value of the discharge current I_d on the current in the electromagnet I_m.

2. Structure choice of the SPT mathematical model

The mathematical model of SPT100 is offered to be represented as a combination of several models reflecting different aspects of the physical processes in the SPT. In particular, the first component of the model is a model determining dependence of the average value of the discharge current $\bar{I}_d$ on the discharge voltage V_p, the current in the electromagnet I_m and the mass flow $\dot{m}$ (1). This model is presented in two equations, each of which corresponds to a separate segment of the range of possible values of the current in the electromagnet:

$$\begin{cases} \bar{I}_d = \bar{I}_{d0}(V_d,\dot{m}) + \dfrac{K_{d1}(V_d,\dot{m})\cdot\alpha_{d1}(V_d,\dot{m})}{(I_m - q_{d1})^2 + \alpha_{d1}(V_d,\dot{m})}, I_m < q_{d2}(V_d,\dot{m}); \\ \bar{I}_d = \bar{I}_{d0}(V_d,\dot{m}) + \dfrac{K_{d1}(V_d,\dot{m})\cdot\alpha_{d1}(V_d,\dot{m})}{(I_m - q_{d1})^2 + \alpha_{d1}(V_d,\dot{m})} + \dfrac{K_{d2}\cdot\left[I_m - q_{d2}(V_d,\dot{m})\right]^2}{\left[I_m - q_{d2}(V_d,\dot{m})\right]^2 + \alpha_{d2}}, \\ I_m \geq q_{d2}(V_d,\dot{m}) \end{cases} \quad (1)$$

where $\bar{I}_{d0}(V_d,\dot{m})$ is the constant component of the average discharge current; $K_{d1}(V_d,\dot{m})$ is the amplitude value of the first bell-shaped function; K_{d2} is the amplitude value of the second bell-shaped function; q_{d1} is the minimal value of the current in the electromagnet in the experiment; $q_{d2}(V_d,\dot{m})$ is the value of the electromagnet current corresponding to the local minimum of the average discharge current; $\alpha_{d1}(V_d,\dot{m})$ and α_{d2} are parameters determining the slope of the bell-shaped functions.

The second component of the SPT model is a model reflecting dependence of the amplitude of oscillation of the discharge current I_p on the discharge voltage V_d, the electromagnet current I_m and the mass flow $\dot{m}$, presented as (2). Symbols in (2) are analogous to those in (1). Functions of two variables in the equation (2) are regression models and were obtained similarly to the functions in (1).

$$\begin{cases} I_p = I_{p01}(V_d,\dot{m}) + \dfrac{K_{p1}(V_d,\dot{m})\cdot\left[I_m - q_{p1}(V_d,\dot{m})\right]^2}{\left[I_m - q_{p1}(V_d,\dot{m})\right]^2 + \alpha_{p1}(V_d,\dot{m})}, I_m \leq q_{p1}(V_d,\dot{m}); \\ I_p = L_1(V_d,\dot{m})\cdot I_m + L_2(V_d,\dot{m}), q_{p1} < I_p < q_{p2}; \\ I_p = I_{p02}(V_d,\dot{m}) + \dfrac{K_{p2}(V_d,\dot{m})\cdot\left[I_m - q_{p2}(V_d,\dot{m})\right]^2}{\left[I_m - q_{p2}(V_d,\dot{m})\right]^2 + \alpha_{p2}(V_d,\dot{m})}, I_m \geq q_{p2}(V_d,\dot{m}), \end{cases} \quad (2)$$

The third component of the static model of the SPT reflects the frequency properties of the discharge current oscillations. For modeling the frequency spectra of the discharge current oscillations obtained experimentally and put into the foundation of the model, a model of a shaping filter is used with white noise supplied as its input. In the experiment, oscillation spectra of the discharge current for different values of the electromagnet current and the optimal structure and parameters of the output filter of the power supply unit were obtained [2, 3]. From the shape of the obtained spectra, the structure of the shaping filter was chosen and its parameters for various values of the electromagnet current were determined. The range of the model operation frequencies is from 0 to 120 kHz, which includes main resonance frequencies of the object, and when the electromagnet current I_m is varied, this range remains constant.

The numeric values of the gain factor of the shaping filters were determined by the value of I_p calculated from equations (2). A typical mathematical model of the shaping filter written in the form of a transfer function looks as follows:

$$W_{Im=3,2}(s) = \frac{I_p(s)}{\xi(s)} = \frac{\sum_{i=0}^{n} b_i s^{n-i}}{\sum_{j=0}^{p} c_j s^{p-j}}, n = 6, p = 8, \quad (3)$$

where s is the Laplace variable, $\xi(s)$ is the Laplace transform for the white noise supplied to the input of the shaping filter; b_i and c_j are parameters of the shaping filter determined in the process of identification on the basis of the experimental data.

3. Results of the SPT mathematical model development

The results of the model are shown as plots in figures 2 and 3. Fig. 2 shows the results of simulation of static characteristics

$$I_p = f_1(V_d, I_m, \dot{m});$$
$$I_d = f_2(V_d, I_m, \dot{m}).$$

The obtained model is represented as functions of three variables, which can be used for development and simulation of automatic control system of the SPT output variables, which is an extremal system. The obtained functions have a strongly manifested extremal nature, and the extremal control system should ensure maintenance of such values of the SPT parameters, with which the amplitude of oscillations and the average value of the discharge current at a given mass flow of the propellant will be at minimum.

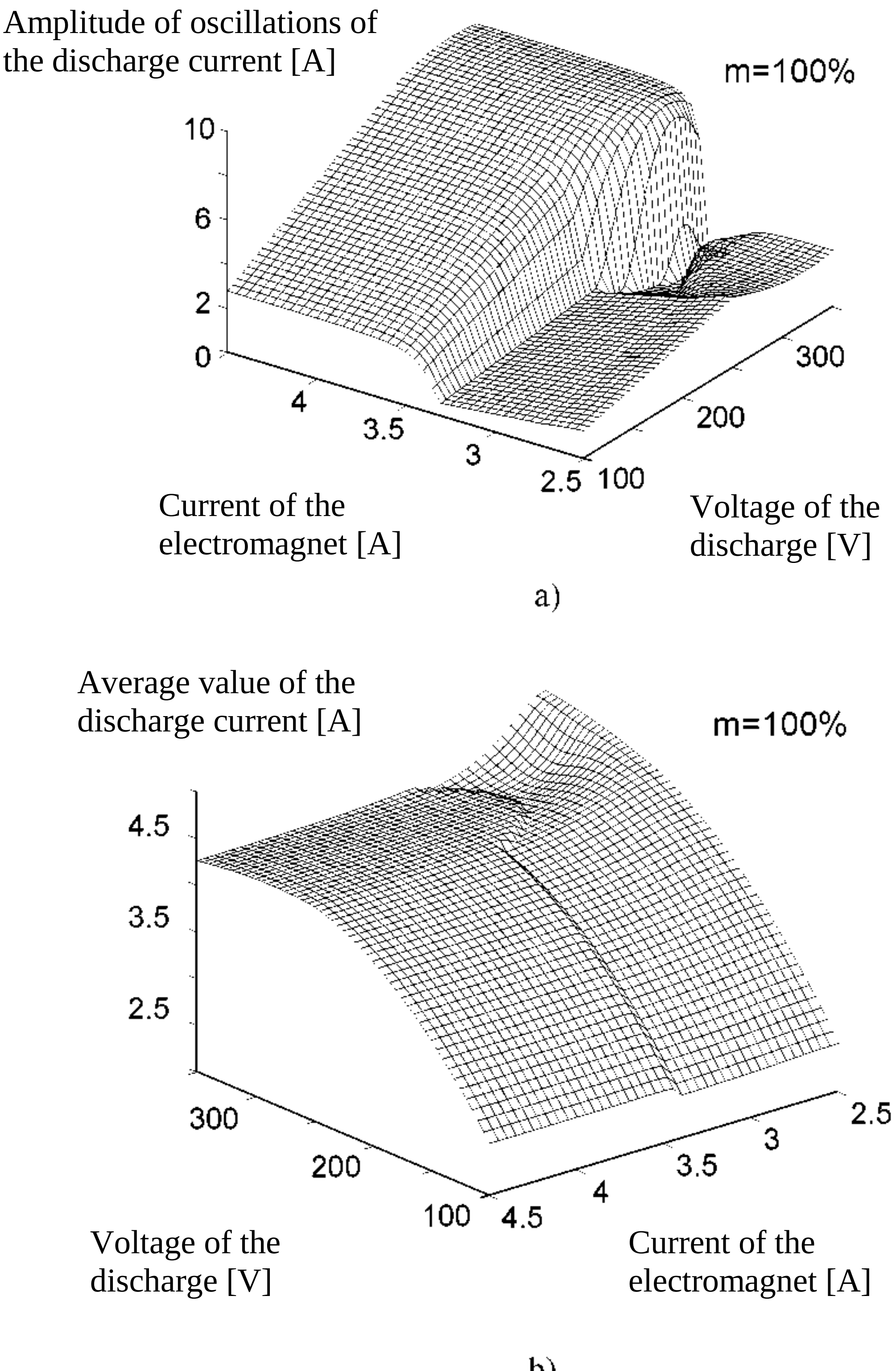

Fig. 2. Outcomes of simulation of the dependence of (a) the amplitude of oscillations and (b) the average value of the discharge current of SPS-100.

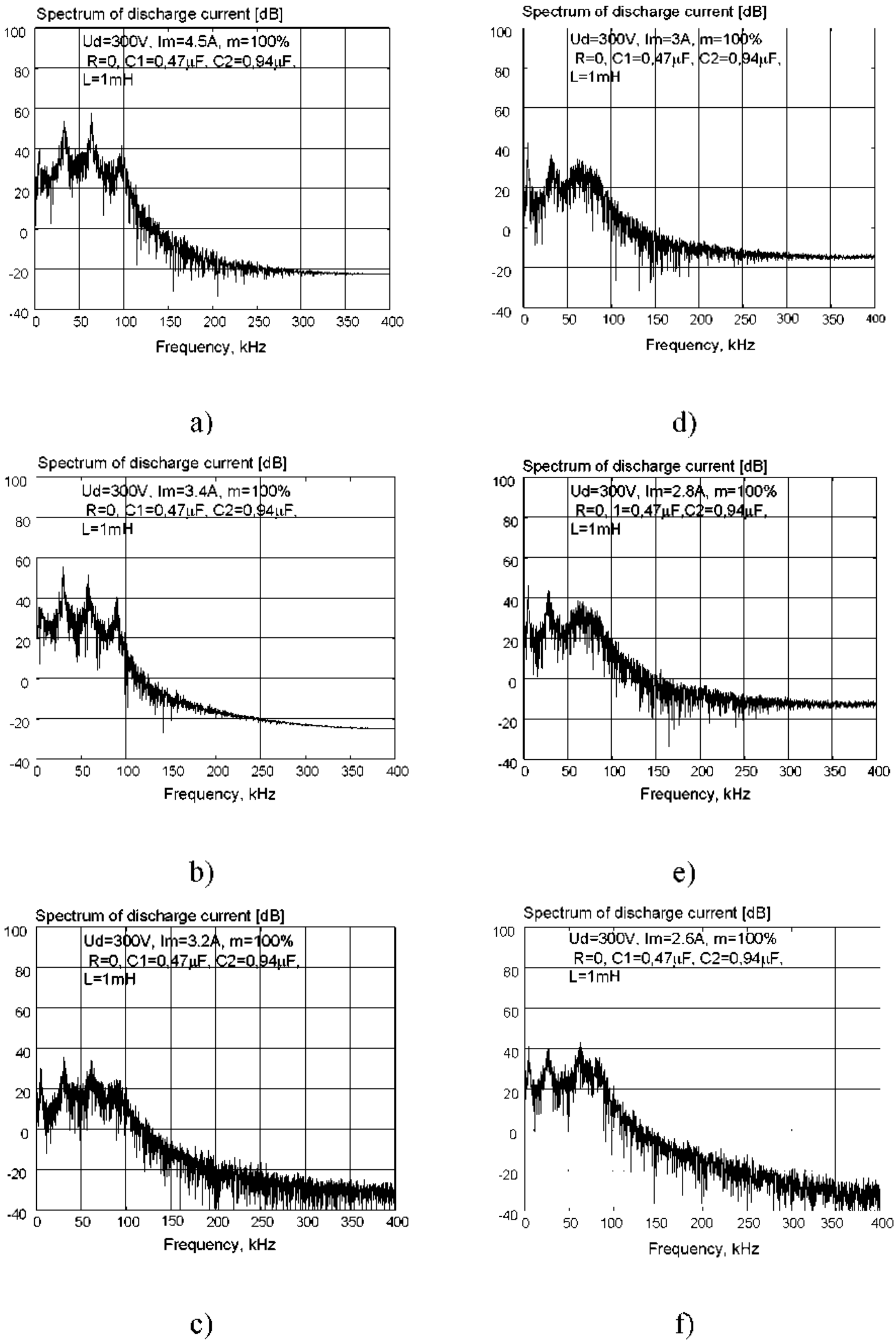

Fig. 3. Dependence of the SPT100 discharge current oscillations spectra

Fig. 3 shows the results of oscillation spectra of the discharge current modeling for different values of the electromagnet current, on the basis of the proposed mathematical model of the SPT. Comparison of the results of simulation with experimental data given in [3, 4] show good match, which confirms adequacy of the obtained mathematical model.

References

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